

# Spin Hall current and two-dimensional magnetic monopole in a Corbino disk

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A spin Hall current can circulate in a Corbino disk with spin-orbit interactions in the presence of a radial charge current, without encountering the sample edge problem. We have suggested several experiments to directly detect and manipulate the spin Hall current in this system. A fictitious two-dimensional magnetic monopole can be created in a Corbino disk with the Rashba interaction by a circular charge current induced either by a spin Hall current in the presence of a ferromagnetic junction or by a perpendicular external magnetic field.

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Recently great attention has been paid to the generation and manipulation of spin currents in semiconductors.[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] A fundamentally interesting and technically promising proposal is that a spin current can be created by the spin Hall effect,[1, 2, 3] arising purely from the relativistic spin-orbit interactions. This can avoid the loss of the coherence of polarized spin current injected from a ferromagnetic metal at the interface due to the conductance mismatch.[11] When a metallic sample is exposed to an external magnetic field, the Lorentz force acting on the current carriers gives rise to a transverse voltage. Similar to the charge accumulation in the conventional Hall effect, spin accumulation is expected at the sample edges in the spin Hall effect. This spin accumulation has been observed recently by Kato *et al.*[12] and by Wunderlich *et al.*[13]. The internal magnetic field [14] and the spin polarization [15] induced by the spin-orbit coupling were also observed in bulk semiconductors. However, as the spin current cannot circulate within the devices measured, the spin Hall current itself has not been observed directly. It is also controversial whether the observed spin accumulation results from intrinsic or extrinsic spin Hall effects.[12, 16]

In this paper, we propose to use a radial current to generate a circulating spin Hall current in a Corbino disk. This would allow the spin Hall current to be directly detected and manipulated. Furthermore, it will be shown that a fictitious two-dimensional magnetic monopole can be created in this Corbino disk system, resulting from the interplay between the spin-orbital interaction and a transverse charge current induced either by a spin Hall current in the presence of a ferromagnetic junction or by a perpendicular external magnetic field. Topologically, a Corbino disk is equivalent to a cylindric system if we take the inner and outer boundaries of the Corbino disk as the two ends of the cylinder. Thus with proper changes of boundary conditions most of the results given below can be also applied to a cylindric system.

A Corbino disk is an annular region of conducting materials surrounding a metallic contact and surrounded by a second metallic contact (Fig. 1a). It has played an im-

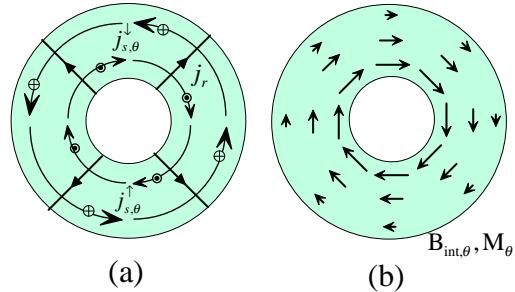


FIG. 1: (a) Corbino disk with an applied radial charge current  $j_r$  and an induced circular spin Hall current polarized perpendicular to the disk plane between the two contacts. The spin Hall current can be taken as a combination of a current of up-spin electrons in the clockwise direction  $j_{s,\theta}^+$  and that of down-spin electrons in the opposite direction  $j_{s,\theta}^-$ . (b) The vortex-like internal magnetic field  $B_{int,\theta}$  or spin polarization  $M_\theta$  generated by the applied charge current in a Rashba system.

portant role in the thought experiments that first clarified the nature of the integer quantum Hall effect[17, 18]. If a radial current  $I$  is applied between the two contacts, it will generate an electric field along the direction of current

$$E_r(r) = \frac{I}{2\pi r w \sigma_0}, \quad (a \leq r \leq b), \quad (1)$$

where  $\sigma_0$  is the longitudinal conductance at zero magnetic field,  $w$  is the thickness,  $a$  and  $b$  are the inner and outer radii of the Corbino disk, respectively. The corresponding voltage drop between the two contacts is

$$V_0 = \frac{I}{2\pi w \sigma_0} \ln \frac{b}{a}. \quad (2)$$

In a spin Hall Corbino disk, a radial electric field will generate a transverse spin current with spins polarized perpendicular to the disk plane. This spin current can circulate persistently within the disk, powered by the radial current  $I$ . The circular spin current density  $j_{s,\theta}$  is

proportional to  $E_r$ . Assuming  $\sigma_{s,\theta}$  to be the spin Hall conductance, induced either by intrinsic or by extrinsic Hall effects, then  $j_{s,\theta}$  is given by

$$j_{s,\theta}(r) = \sigma_{s,\theta} E_r(r) = \frac{\sigma_{s,\theta} I}{2\pi w r \sigma_0}. \quad (3)$$

The corresponding total spin current is

$$I_s = d \int_a^b dr j_{s,\theta}(r) = \frac{\sigma_{s,\theta} I}{2\pi \sigma_0} \ln \frac{b}{a}. \quad (4)$$

This spin current can be taken as a combination of a current of clockwise-circulated up-spin electrons and a current of counterclockwise-circulated down-spin electrons. It results in a flow of spin angular momentum without net charge current.

In the ordinary Hall effect, a transverse force perpendicular to an electric field is provided by the Lorentz force from a magnetic field. In a spin Hall system, this transverse force arises from the relativistic spin-orbit interaction. The spin-orbit coupling does not depend on a magnetic field. It opens a new route to the manipulation of spins with non-magnetic semiconductors without external magnetic fields.

In a two-dimensional electron gas of semiconductors, the spin-orbit interaction arising from the asymmetry in the growth direction ( $z$ ) has a standard Rashba form.[19] Recently, this kind of spin-orbit interaction has attracted special attention, in part due to the work of Sinova *et al.* who showed that an intrinsic and universal spin Hall current can be induced by this interaction.[2] The Hamiltonian of an electron gas with the Rashba spin-orbit coupling is defined by

$$\hat{H} = \sum_k \left( \frac{\hbar^2 k^2}{2m} - \alpha_R \mathbf{e}_z \cdot \mathbf{k} \times \boldsymbol{\sigma} \right), \quad (5)$$

where  $\mathbf{k}$  is the wavevector and  $\boldsymbol{\sigma}$  is the Pauli matrix. The Rashba interaction couples the orbital motion of electrons with their spins. Effectively, it can be taken as a Zeeman coupling of a spin subjected to the interaction of an internal magnetic field. Thus the above Hamiltonian can be also expressed as

$$\hat{H} = \sum_k \left[ \frac{\hbar^2 k^2}{2m} - \frac{1}{2} g \mu_B \mathbf{B}_{\text{int}}(\mathbf{k}) \cdot \boldsymbol{\sigma} \right], \quad (6)$$

where  $\mu_B$  is the Bohr magneton,  $g$  is the gyromagnetic ratio, and

$$\mathbf{B}_{\text{int}}(\mathbf{k}) = \frac{2\alpha_R}{g\mu_B} \mathbf{e}_z \times \mathbf{k} \quad (7)$$

is the effective internal magnetic field.

Without an external electric field, the internal field  $\mathbf{B}_{\text{int}}$  in real space is zero since the average of the momentum  $\mathbf{k}$  is zero over the Fermi sea. However, by applying an electric field to the system, the average of  $\mathbf{k}$  becomes

finite in the direction opposite to the applied field. In this case a finite magnetic field perpendicular to both the applied electric field and the disk plane will be generated by the Rashba spin-orbital coupling. Thus for the system considered here, a radial electric current will induce an internal magnetic field in the radian direction. Under the semiclassical approximation, it can be readily shown that this internal magnetic field is given by  $\mathbf{B}_{\text{int}}(\mathbf{r}) = B_{\text{int},\theta}(r) \mathbf{e}_\theta$  and

$$B_{\text{int},\theta}(r) = -\frac{A}{2\pi r}, \quad (8)$$

where  $A = (2e\tau\alpha_R I)/(\hbar g\mu_B w\sigma_0)$  and  $\tau$  is the relaxation time of electrons. As in the usual electron gas system, this internal magnetic field will polarize the electrons and lead to a finite spin magnetization at each point parallel to  $\mathbf{B}_{\text{int}}(r)$ . From the spin susceptibility of a two-dimensional electron gas and Eq. (8), it can be shown that the spin magnetization induced by the Rashba interaction is proportional to  $\mathbf{B}_{\text{int}}$  and given by

$$\mathbf{M}(\mathbf{r}) = \frac{1}{2} g \mu_B \langle \boldsymbol{\sigma} \rangle = -\frac{e\alpha_R m \tau g \mu_B}{4\pi \hbar^3} E_r \mathbf{e}_\theta. \quad (9)$$

Fig. (1b) shows the spatial distribution of  $\mathbf{B}_{\text{int}}$  or equivalently  $\mathbf{M}$ . The spatial dependence of  $\mathbf{B}_{\text{int}}(\mathbf{r})$  is similar to the magnetic field generated by a line current along the  $z$ -axis at  $r = 0$ . But  $\mathbf{B}_{\text{int}}$  exists only inside the sample. This kind of internal magnetic field as well as the spin polarization induced by the spin-orbit coupling were observed in bulk semiconductors by the time and spatially resolved Faraday or Kerr rotation.[12, 14, 15]

The Rashba coefficient  $\alpha_R$  can be tuned over a wide range by a vertical electric field or a strain pressure.[20] In bulk semiconductors without strain, the internal magnetic field produced by an applied electric current and the Rashba interaction is generally less than  $10^{-6}\text{T}$  and difficult to be detected experimentally[12]. However, by adding pressure or strain to the sample, the Rashba coupling  $\alpha_R$  is dramatically enhanced and the internal magnetic field can be as high as  $10^{-3}\text{T}$ .[12] Thus in a Corbino disk of strained semiconductor, the predicted spatial distribution of the internal field (8) or spin polarization (9) can be easily detected by the Kerr rotation experiment. The induced spin polarization can be also probed directly by scanning SQUID (superconducting quantum interference device) measurements.

In a pure Rashba system, the longitudinal conductivity  $\sigma_0$  is given by

$$\sigma_0 = \frac{e^2 n \tau}{m} - \frac{me^2 \tau}{2\pi \hbar^4} \alpha_R^2, \quad (10)$$

where  $n$  is the electron concentration. The first term is the the usual Drude term. The second term is the correction to the Drude conductivity from the Rashba spin-orbit interaction. In real materials, there are always impurities. The spin-orbit interaction resulting from the gradient of the impurity scattering potential can affect

both the spin Hall effect and the charge dynamics of the system. As shown by Hirsch[3], the skew scattering of a spin current by impurity spin-orbit interaction can lead to a transverse charge imbalance, giving rise to a Hall voltage. This will add an impurity dependent term to Eq. (2) and change the voltage between the two contacts from  $V_0$  to

$$V = \left(1 - \frac{4\pi R_s g \mu_B e n \sigma_{s,\theta}}{\hbar}\right)^{-1} V_0, \quad (11)$$

where  $R_s$  is the anomalous Hall coefficient. If the anomalous Hall effect is induced purely by impurity scattering in this paramagnetic system, then  $R_s$  is expected to be proportional to the impurity concentration. Experimental measurement of this impurity-induced correction to the voltage fall between the inner and outer contacts will provide a direct detection of the circulated spin current. Furthermore, by varying the Rashba coupling, for example, by applying a vertical electric field or a strain pressure, one can also determine whether the spin Hall current is mainly generated by the intrinsic spin-orbit coupling or by the extrinsic impurity scattering from this kind of measurements.

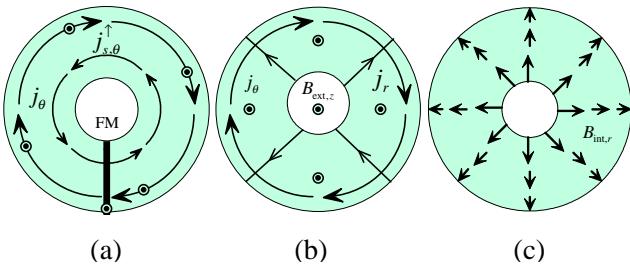


FIG. 2: (a) Corbino disk with a narrow ferromagnetic junction polarized along the  $z$  direction. The width of the junction is assumed to be smaller than the spin diffusion length. An intrinsic charge Hall current can be generated by the spin Hall effect if only up-spin electrons can tunnel through the junction. (b) Corbino disk in a perpendicular external magnetic field. Besides the applied current  $j_r$  between the inner and outer contacts, there is also a transverse Hall current  $j_\theta$  maintained by the Lorentz force. (c) The internal magnetic field induced by the charge Hall current in the disk with Rashba spin-orbit interactions. Effectively, this radial magnetic field can be taken as the field created by a two-dimensional magnetic monopole at  $r = 0$ . But  $B_{int,r}$  is finite only inside the disk.

Direct measurement of the spin Hall current in the Corbino disk is difficult since the spin current does not couple with electromagnetic fields. However, one can measure the charge current generated by the spin Hall effect if one can use a narrow ferromagnetic (FM) junction to block the down-spin Hall current and allow only the up-spin Hall current to flow inside the disk (Fig. 2a). This requires that the FM junction is magnetically polarized along the  $z$ -direction and the width of the junction

is smaller than the spin diffusion length. In this case, only up spin electrons can tunnel through the junction and down spin electrons will accumulate by the one side of the junction facing the direction of the down-spin current.

The accumulation of down spin electrons will form effectively a potential barrier to prevent the tunnelling of up spin electrons through this accumulation area. Two limiting situations can happen depending on the degree of accumulation. One is the strong accumulation limit at which the up-spin Hall current is completely blocked by the potential barrier. The other is the weak accumulation limit at which the potential barrier is low and the up-spin electrons can tunnel through this accumulation barrier to form a persistently Hall current inside the disk.

The weak accumulation limit is physically more interesting. In this limit, a Hall current of up spin electrons will flow in the disk. This is a charge Hall current entirely induced by the spin Hall effect, without any external magnetic field. This Hall current density  $j_\theta$  is expected to be proportional to the spin Hall current  $j_{s,\theta}$  determined by Eq. (3), i.e.

$$j_\theta(r) = -\frac{2e\delta}{\hbar} j_{s,\theta}(r), \quad (12)$$

where the dimensionless coefficient  $\delta$  is expected to be smaller than  $1/2$ .  $j_\theta(r)$  can be measured experimentally. An experimental confirmation of this transverse charge current without external magnetic fields would be a direct and unambiguous proof of the spin Hall effect.

As for  $j_r$ , the interplay between the Rashba interaction and the Hall current  $j_\theta$  will also induce an internal magnetic field in the direction perpendicular to  $j_\theta$  inside the disk. Thus in this case the internal magnetic field contains not only the  $B_{int,\theta}(r)$  term induced by  $j_r$  (Eq. (8)), but also a radial term  $B_{int,r}$  induced by  $j_\theta$ , i.e.  $\mathbf{B}_{int}(\mathbf{r}) = B_{int,r}(r)\mathbf{e}_r + B_{int,\theta}(r)\mathbf{e}_\theta$ . Within the semiclassical approximation,  $B_{int,r}$  is given by

$$B_{int,r}(r) = -\frac{2e\tau\alpha_R j_\theta(r)}{\hbar g \mu_B \sigma_0} = \frac{e_g}{2\pi r}, \quad (13)$$

where  $e_g = (2eA\delta\sigma_{s,\theta}^z)/(\hbar\sigma_0)$ . This is a divergent magnetic field. It is like the field generated by a two-dimensional magnetic monopole with a “charge” proportional to  $e_g$ . A schematic illustration of the distribution of  $B_{int,r}(r)$  is shown in Fig. (2c). If we set the inner radius  $a \rightarrow 0$ , then it is straightforward to show that the divergence of  $\mathbf{B}_{int}$  is given by

$$\nabla \cdot \mathbf{B}_{int} = e_g \delta(\mathbf{r}). \quad (14)$$

This is a fundamentally interesting result. It indicates that one can create a fictitious magnetic monopole in a Corbino disk with the Rashba spin-orbit interaction simply by applying a circular current. This fictitious magnetic monopole results from the interplay between the complex topology of the Corbino disk and the spin-orbit interaction. We believe that a multiple-fictitious-magnetic-monopole system can be similarly created in a

more complex topological system. This would provide an opportunity to simulate experimentally the Maxwell equations in the presence of magnetic monopoles.

One can also create a charge Hall current in the disk without the FM junction, but by applying a vertical magnetic field  $B_{\text{ext},z}$ , as shown in Fig. (2b). Since the Lorentz force produced by the external magnetic field acts symmetrically on both the up and down spin electrons, now the charge Hall current is not spin polarized. From the standard theory of electromagnetic response, we know that the Hall current density is given by

$$j_\theta(r) = \frac{\omega_c \tau I}{2\pi w r}, \quad (15)$$

where  $\omega_c = eB_{\text{ext},z}/m$  is the cyclotron resonance frequency. As for the case shown in Fig. (2a), this circular current will also induce an internal magnetic field along the radial direction. But now  $B_{\text{int},r}$  becomes

$$B_{\text{int},r}(r) = \frac{\omega_c \tau A}{2\pi r}. \quad (16)$$

It can be also taken as a magnetic field generated by a fictitious two-dimensional magnetic monopole. The “charge” of the monopole is now proportional to  $\omega_c \tau A$ .

In conclusion, a spin Hall current can circulate in a Corbino disk in the presence of a radial charge current. Experimental measurements of the voltage drop between the two contacts as functions of  $\alpha_R$  and  $R_s$  would allow us to judge unambiguously if the spin current is mainly produced by the intrinsic Hall effect.  $\alpha_R$  and  $R_s$  can be tuned by applying a vertical electric field or a strain pressure and by altering the impurity concentration, respectively. The interplay between the Rashba spin-orbital interaction and a circular charge current, which can be created either by a spin Hall current in the presence of a narrow ferromagnetic junction or by a perpendicular external magnetic field, will induce a divergent internal magnetic field. This divergent internal magnetic field can be effectively taken as the field produced by a fictitious two-dimensional magnetic monopole. Thus the Corbino disk with spin-orbit interactions provides a real-space realization of the Dirac magnetic monopole in a solid state system. It can be used as a basic unit for further studies of Maxwell electrodynamics with magnetic monopoles.

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